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SOLUTION OF A PROBLEM.

BY R. J. ADCOCK, MONMOUTH, ILLINOIS.

Problem.—The velocity of the wind being v, and its force being proportional to the square of the velocity; and turning a wind wheel with a horizontal axis, the driving surface of the wings being described by a straight line perpendicular to, and moving uniformly along, the axis, while it rotates uniformly about the axis; what then must be the inclination to the plane, perpendicular to the axis, of the part of the wing farthest from the axis, in order that the wheel when moving with the proper angular velocity, may be doing a maximum amount of work in a given time?

Solution.—Let $f =$ the force of the wind on a unit of surface when moving with a unit of velocity, then $fv^2 =$ its force for velocity v.

Let $b =$ dist. moved along axis by gen. line of wing surf. in one revol'n,
$x =$ distance of any point of generatrix from axis,
$z =$ angle made with plane perp. to axis by helix at distance x,
$z_1 =$ same at distance r, the length of generatrix,
$a =$ angular velocity of rotation of wheel. Then

$$b = 2\pi x \tan z = 2\pi r \tan z_1,$$

$$\sqrt{(b^2+4\pi^2x^2)}dx = -\frac{b^2dz}{2\pi\sin^3 z} = \text{elementary area of wings,}$$

$v - ax\tan z =$ velocity with which the wind strikes this elementary surface,

$$-f\sin z\frac{\cos^2 z}{\sin z}(v \quad ax\tan z)^2\frac{b^3dz}{4\pi^2\sin^3 z} = -f(v-ax\tan z)^2\frac{b^3\cos^2 z dz}{4\pi^2\sin^3 z} = \text{elementa-}$$

ry moment; and

$$-fa(v - ax\tan z)^2\frac{b^3\cos^2 z dz}{4\pi^2\sin^3 z} = \text{elementary work done in a unit of time;}$$

which in case of a maximum for a, gives

$$(v - ax\tan z)^2 - 2a(v - ax\tan z)x\tan z = 0,$$

$$v - ax\tan z = 0,\ v - ax\tan z - 2ax\tan z = 0,\ ax\tan z = \tfrac{1}{3}v.$$

Since, from (1), $x\tan z = r\tan z_1$, $ar\tan z_1 = \frac{1}{3}v$. Integrating elementary area,

$$-\int_{\frac{\pi}{2}}^{z_1}\frac{b^2dz}{2\pi\sin^3 z} = \pi r^2\tan z_1\left(\frac{\cos z_1}{\sin^2 z_1} - \log\tan\tfrac{1}{2}z_1\right) = \text{entire area.}$$

By reduction the elementary moment is

$$-f(v - ax\tan z)^2\frac{b^3\cos^2 z dz}{4\pi^2\sin^3 z} = -\tfrac{1}{9}fv^2\frac{b^3\cos^2 z dz}{\pi^2\sin^3 z} = -\tfrac{8}{9}fv^2\pi r^3\tan^3 z_1\frac{\cos^2 z dz}{\sin^3 z}.$$

And the entire moment when moving with proper angular velocity is

$$\int_{\frac{\pi}{2}}^{z_1} -\tfrac{8}{9} fv^2 \pi r^3 \tan^3 z_1 \frac{\cos^2 z dz}{\sin^3 z} = \tfrac{4}{9} \pi fv^2 r^3 \tan^3 z_1 \left(\frac{\cos z_1}{\sin^2 z_1} + \log \tan \tfrac{1}{2} z_1\right).$$

Hence, $\dfrac{\frac{4}{9} fv^2 r \tan z_1 [(\cos z_1 \div \sin^2 z_1) + \log \tan \frac{1}{2} z_1]}{(\cos z_1 \div \sin^2 z_1) - \log \tan \frac{1}{2} z_1} = \text{a maximum for } z_1,$ (2)

will give the position of wings when the work being done is a maximum.

Therefore, placing the first differential coefficient $= 0$, and solving for $\log \tan \frac{1}{2} z_1$, there results

$$\log \tan \tfrac{1}{2} z_1 = (\cos z_1 \div \sin^2 z_1)[1 + \cos^2 z_1 \pm \sqrt{(4 + \cos^4 z_1)}].$$

Substituting in (2) this second value, the only one applicable,

$$\frac{\tan z_1 [1 + 1 + \cos^2 z_1 - \sqrt{(4 + \cos^4 z_1)}]}{1 - 1 - \cos^2 z_1 + \sqrt{(4 + \cos^4 z_1)}} = \frac{\tan z_1 [2 + \cos^2 z_1 - \sqrt{(4 + \cos^4 z_1)})}{\sqrt{(4 + \cos^4 z_1)} - \cos^2 z_1}$$

$$= -\tan z_1 + \tfrac{1}{2} \tan z_1 [\cos^2 z_1 + \sqrt{(4 + \cos^4 z_1)}] = \text{maximum for } z_1.$$

Differentiating a second time and reducing,

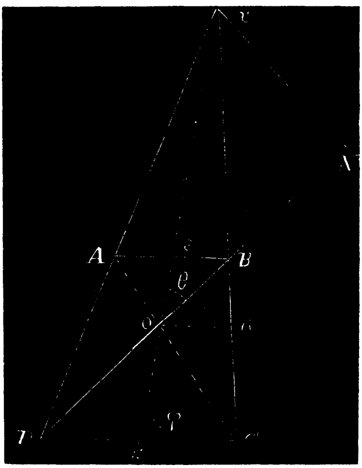
$$\cos^6 z_1 - \tfrac{7}{2} \cos^4 z_1 - 2 \cos^2 z_1 + 2 = 0, \text{ from which}$$

$$\cos^2 z_1 = 0.55154\,;\ \therefore z_1 = 42^\circ 2\tfrac{1}{2}'.$$

SOLUTION OF A PROBLEM.

BY PROF. C. M. WOODWARD, WASHINGTON UNIV., ST. LOUIS, MO.

Problem.—Given a frustrum of an oblique cone with a circular base; the frustrum is cut in two by a plane perpendicular to the principal plane of the cone, and tangent to the two bases. Find the ratio of the volumes of the two parts of the frustrum.



Solution.—Let $vABCD$ be the section of the cone made by the principal plane, and let DBN be the trace of the intersecting plane perpendicular to it. The section, projected in DB, is obviously an ellipse of which DB is the major axis. Let $ov = h$, $As = r$, $Dk = R$ and $on = x$. Then

$$DB = (R + r)\frac{\sin \varphi}{\sin \theta}.$$

The minor axis is the chord of a circle whose radius is $\frac{1}{2}(R + r)$, and whose dist. from the center is $\frac{1}{2}(R - r)$; its length is therefore $2\sqrt{(Rr)}$.